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On the Mean Distances of the Groombridge Stars.

By A. S. Eddington, B.A. M.Sc.

In a former paper * I have given some calculations as to the mean distance of the stars of Groombridge's catalogue for the purpose of comparing the distances of the two star-drifts, with which that paper dealt. The results obtained were, I believe, sufficient to show that the two drifts must be at, roughly, the same mean distance from us, but there seemed no means of judging how closely the actual numerical results could be relied on. Some further reflection has, however, shown that the method has advantages (as regards freedom from systematic error) which were not at that time noticed; it also incidentally furnishes an additional test of the two-drift hypothesis. I have accordingly developed the results more precisely in this paper.

The principle of the method of determining the mean parallax of a large group of stars is well known. It depends on determining the apparent angular displacement corresponding to a known linear displacement. For example, in Professor Kapteyn's researches the parallactic motion has been mainly used as the known linear displacement; if that is assumed to be the same for all the different groups of stars discussed, the corresponding angular motion is proportional to the mean parallax of the group. In the present case the mean peculiar speed (*i.e.* the mean individual motion, irrespective of direction, which remains after all drift or parallactic motion has been abstracted) is taken for a similar purpose. If its linear amount is assumed to be the same in all parts of the heavens, the angular amount will be proportional to the mean parallax of the stars. We must, of course, be prepared for the possibility that neither the parallactic motion nor the mean peculiar speed is strictly constant in different parts of the sky or for different classes of stars. The determinations of "hV" (or the ratio of the drift motion to the mean peculiar speed) made in the previous paper throw some light on this question; they appear to favour the assumption of constant mean peculiar speed, but the test is at present a rather rough one.

As the proper motions of the stars include not only their peculiar motions but also the drift-motions, the mean peculiar motion cannot be found without mathematical investigation. The theory developed in the previous paper enabled this to be done. There are two stages in the calculation:—(1) The constants of the drifts must be determined from the numbers of stars moving in the different directions (without regard to the magnitudes of their proper motions); and (2) from these constants the theoretical relation between the mean peculiar motion and the whole mean motion can be found.

For example, consider that division of the sky which I have

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* *Monthly Notices,* vol. lxvii. p. 34, referred to hereafter as "*Systematic Motions.*" The portion especially dealing with mean distances is pp. 55–57.

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called Region B. In fig. 1, A is for this region the theoretical curve of the kind considered in the former paper, in which the radius vector is proportional to the *number* of stars moving in the corresponding direction; B is the theoretical curve in which the radius vector is proportional to the *mean proper motion* of the stars moving in the corresponding direction. The two curves are rather similar; the bi-lobed character of B is readily recognised, although the elongations and minima are much less pronounced than in A.

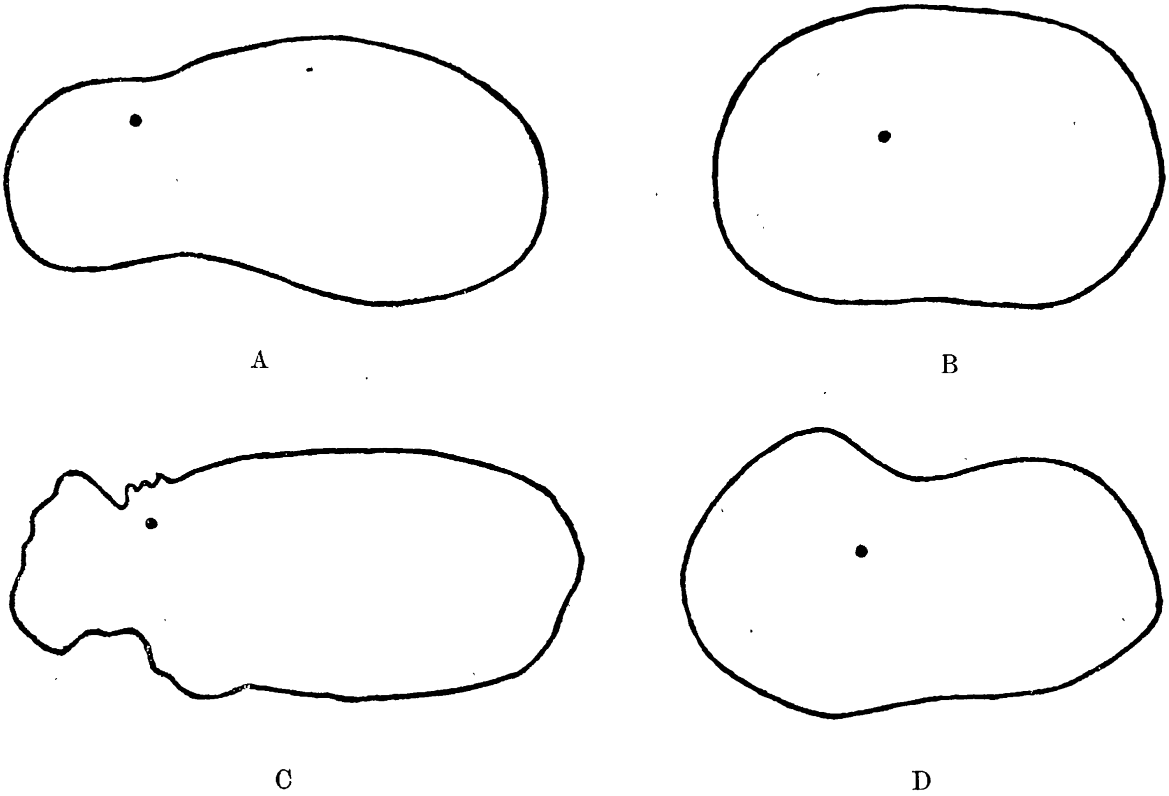

Fig. 1.—Diagrams for Region B.

A and C—theoretical and observed curves, showing total number of proper motions in different directions.

B and D—theoretical and observed curves, showing mean proper motion in different directions.

It should be mentioned that in drawing B the two drifts have been assumed to be at the same mean distance; as the two lobes of the curves correspond respectively to stars of the two drifts, it is clear that if one of the drifts is nearer than the other, this will be shown by the lobe of the curve B, which corresponds to that drift, being exaggerated in size compared with the other. It is in this way that the mean distances of the drifts can be compared. The curves C and D are those derived from observation, and are to be compared respectively with the theoretical curves A and B. Although our main purpose in this investigation is to discuss the distance of the stars of the two drifts, attention may be called to

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the clear way in which, in this and in other regions, the mean proper motions support the two-drift hypothesis. The evidence of the observed curves C and D respectively in favour of that hypothesis must be regarded as, at least to a large extent, independent; it could hardly be expected, *a priori*, that the mean proper motions would be greatest in the two directions in which the total number of proper motions is greatest. It may be noticed that the two drifts are slightly more prominent in the observed curve D than in the theoretical curve B; some of the other regions considered show this rather more strongly, and it appears to be a general result. I have not been able to arrive at any satisfactory explanation of it.

For the other regions, reference must be made to Table I. In it will be found, for each of the seven regions covered by the Groombridge catalogue, the mean proper motion of stars moving in each of twelve directions at intervals of 30°. (For forming the mean proper motion in any direction, stars moving in directions within 15° on either side of it were used; thus each direction corresponds to a 30° sector.) Curves similar to D in fig. 1 could readily be constructed from these data, and we should find that they are clearly bilobed, indicating two drifts. Region D constitutes an exception; for, since it contains the apex of Drift II., that drift is not very apparent in it. The same thing may be seen by inspecting the numbers in the table: in each column there are two maxima and two minima well indicated, and a comparison with the Tables II. to VII. in "*Systematic Motions*" shows that they correspond with the maxima and minima in the numbers of stars moving in the different directions.

TABLE I.

Mean Proper Motions (rejecting highest eighth).

Direction θ.		Region.						
Regions A and B	C D E F and G	A	B	C	D	E	F	G
°	°	″	″	″	″	″	″	″
5	15	5·65	4·64	2·75	3·23	4·22	2·99	2·29
35	45	3·99	3·37	4·22	3·54	3·85	4·72	2·11
65	75	3·00	2·40	3·80	4·72	5·24	6·76	2·64
95	105	2·74	2·51	2·61	5·04	4·72	3·49	2·93
125	135	3·11	2·57	3·34	5·11	5·86	3·20	2·87
155	165	3·35	2·90	2·82	3·76	6·42	2·94	2·09
185	195	3·32	2·54	2·74	3·15	6·90	4·47	1·70
215	225	4·28	2·21	1·53	2·10	4·95	5·94	1·94
245	255	3·50	2·14	1·94	2·71	4·33	4·88	2·92
275	285	2·49	1·54	1·61	2·07	2·06	4·39	3·53
305	315	3·80	1·46	1·32	2·00	2·77	3·81	4·40
335	345	4·84	3·60	2·50	3·76	5·49	2·98	2·81

In calculating these means, it is necessary to adopt some rule for rejecting excessive proper motions. The difficulty is to find a limit which will affect the two drifts equally. The rule which

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I have adopted, which seems to meet this, has been to reject the fastest eighth moving in each direction. That is to say, if there are n proper motions in one of the twelve 30° sectors, I pick out and reject the $n/8$ highest of these, and take the mean of the remainder. The net result must be nearly the same as if the nearest stars of each drift, to the number of rather more than one-eighth of the whole, were rejected. This means a much more drastic rejection of high proper motions than is usually resorted to; but some such plan seems necessary if the more distant stars are not to be altogether outweighted.

Each of the mean proper motions provides us with an equation involving linearly the unknown mean parallaxes of the two drifts (see "*Systematic Motions,*" p. 56, equation (1)). Thus for each region twelve equations were formed, involving these two unknowns; these were properly weighted and solved by least squares. Table II. contains the results. "d_1" and "d_2" denote the distances of the drifts. For the meaning of "h" reference must be made to the mathematical analysis of the former paper; it is connected with the mean peculiar speed of the stars. It is, however, the relative rather than the absolute values of the distance that are of interest, so that the precise unit used is not of great importance; but, accepting Campbell's provisional value (20 k.m. per sec.) for the speed of the solar motion, and my own determinations of the drift-constants, the actual mean parallaxes may be obtained by dividing the numbers in Table II. by 500. (It must, however, be remembered that a great many near stars have been rejected, as explained above, so that the mean parallax has a rather artificial significance.)

Table II.

Mean Parallaxes.

(The quantities $\frac{1}{hd_1}$ and $\frac{1}{hd_2}$ tabulated are the mean parallaxes multiplied by a factor whose value is probably about 500.)

Region.	*N.P.D.*	R.A.	$\frac{1}{hd_1}$.	$\frac{1}{hd_2}$.
	° °	h h	″ ″	″ ″
A	0–20	0–24	2·96 ± ·07	3·41 ± ·13
B	20–52	22– 2	2·45 ± ·07	2·40 ± ·11
C	,,	2– 6	2·39 ± ·08	2·65 ± ·12
D	,,	6–10	3·35 ± ·12	3·23 ± ·21
E	,,	10–14	3·65 ± ·14	4·78 ± ·29
F	,,	14–18	3·74 ± ·23	4·15 ± ·31
G*	,,	18–22	2·77 ± ·16	2·55 ± ·18

The equality of distance of the two drifts is evidently confirmed. Region E is the only one in which there is any indication of a considerable difference between their distances. The probable errors are of interest, not only as indicating the reliability of the

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* The restricted region G (excluding subdivision γ) was used; see *Systematic Motions*, p. 52.

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results found, but as a measure of the accordance between theory and observation, for they are derived from the discordance of the calculated and observed mean proper motions in the twelve directions. Region B, which was represented in fig. 1, shows the best agreement between theory and observation; but it may be remarked that, where the agreement is not so good, that is always due to the phenomenon mentioned above—that the mean proper motions show the effect of the two drift-motions more prominently than calculation leads us to expect.

Before we can rely on the values of the probable error given in Table II., it is necessary to consider whether a possible systematic error in the catalogue, from which the proper motions are derived, may not lead to errors of importance not included in the theoretical probable error. The conclusion is rather unexpected; a systematic error in the proper motions does not invalidate these results. The reason is this: a systematic error affects the mean proper motion, and also the total number of proper motions, in a particular direction; owing to the latter error, we arrive at incorrect values of the constants of the drifts; but it is not difficult to see that, if these incorrect values of the drift-constants are used in forming the equations of condition, this will compensate for using incorrect values of the mean proper motions. Looked at in another way, the analysis was designed to extricate the mean peculiar motion from the systematic drift-motions with which it is involved; it at the same time extricates it from any possible systematic error which is hardly distinguishable in its effect from systematic drift or parallactic motion. In order that this elimination of error may take place, it is necessary to use drift-constants derived from the observed proper motions in the particular region considered, not those determined from an examination of the whole sky; this has been attended to.

Assuming that the two drifts are actually at the same mean distance, and accordingly combining their parallaxes given in Table II. with weights inversely proportional to the squares of their probable errors, we have the following table of mean parallaxes (multiplied by a factor) of the Groombridge stars in different parts of the sky.

TABLE III.

Variation of the Mean Parallax with Galactic Latitude.

Region.	Mean Galactic Latitude.	Mean Parallax (multiplied by a factor).
	°	″
B	−12	2·43
C	− 2	2·47
G	+ 8	2·67
A	+27	3·06
D	+32	3·32
F	+46	3·88
E	+66	3·87

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The mean parallax steadily increases with the distance from the galaxy, a result which is in accordance with the generally accepted ideas of the distribution of stars, viz. that the increased number of stars in the low galactic latitudes is due to additional more distant stars being visible, and not to any crowding among the nearer stars. The table apparently indicates a *gradual* change in the mean distance; but, as the regions are rather extensive in area, it is not definitely incompatible with the hypothesis that the differences are caused by a belt of distant stars almost limited to the galaxy.

On Ancient Eclipses. By P. H. Cowell, M.A., F.R.S.

I am much obliged to Mr. Crommelin for repeating my calculations in the case of six solar eclipses, and so setting the question of their accuracy beyond doubt.

The fact that emerges from the discussion of the eclipses is that there is an unexpected rate of change in what may be termed the "nodical year," or period of revolution of the Sun relatively to the Moon's node.

This abnormal rate of change may arise from secular changes in the motion of the Moon's node (other than that arising from the change of the eccentricity of the Earth's orbit round the Sun), or from secular changes in the motion of the Sun, or, of course, from a combination of both hypotheses.

The motion of the node, as found from observation, differs very slightly from the formula given for it by Professor Brown in *M.N.*, lxiv. p. 532. This formula is differentiated and the secular acceleration of the node obtained on the supposition that the eccentricity and inclination of the Moon's orbit are constant. We know no reason why these quantities should vary, and observation shows that if they do vary, the variations do not exceed certain limits. It happens, however, that these limits are sufficiently large to admit of a considerable part of the unexpected change in the nodical year being attributed to the motion of the node (at least as an alternative hypothesis), thus diminishing the part to be ascribed to the Sun.

The formula $+4''(\mathrm{T}-18{\cdot}25)^2-1''{\cdot}0$ takes numerical values $+1''{\cdot}0$ in 1755 and 1895, and $-1''{\cdot}0$ in 1825, with lesser values numerically at intermediate dates. A second of arc is not an impossibly large quantity to attribute to systematic errors, or even to unknown long-period terms, and it is clear from Professor Newcomb's table (*Ast. Const.*, p. 22) that residuals of over 1″ cannot be avoided by any formulæ.

Hence it must be understood that a possible change in the eccentricity and inclination of the Moon's orbit is put forward as